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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/22**

Paper 2 Further Pure Mathematics 2 **October/November 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the value of $\int_6^7 \frac{1}{\sqrt{(x-5)^2-1}}\,dx$, giving your answer in the form $\ln(a+\sqrt{b})$, where a and b are integers to be determined. [4]

2 The curve C has equation

$$4y^2 + 4\ln(xy) = 1.$$

(a) Show that, at the point $\left(2, \frac{1}{2}\right)$ on C, $\frac{dy}{dx} = -\frac{1}{6}$. [3]

(b) Find the value of $\dfrac{d^2y}{dx^2}$ at the point $\left(2, \frac{1}{2}\right)$. [4]

3 The curve C has parametric equations

$$x = \frac{1}{2}e^{2t} - \frac{1}{3}t^3 - \frac{1}{2}, \qquad y = 2e^t(t-1), \qquad \text{for } 0 \leqslant t \leqslant 1.$$

Find the exact length of C. [7]

4 (a) Use de Moivre's theorem to show that

$$\cot 6\theta = \frac{\cot^6\theta - 15\cot^4\theta + 15\cot^2\theta - 1}{6\cot^5\theta - 20\cot^3\theta + 6\cot\theta}.$$ [6]

(b) Hence obtain the roots of the equation

$$x^6 - 6x^5 - 15x^4 + 20x^3 + 15x^2 - 6x - 1 = 0$$

in the form $\cot(q\pi)$, where q is a rational number. [4]

5 Find the particular solution of the differential equation

$$3\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x^2,$$

given that, when $x=0$, $y=\frac{dy}{dx}=0$. [10]

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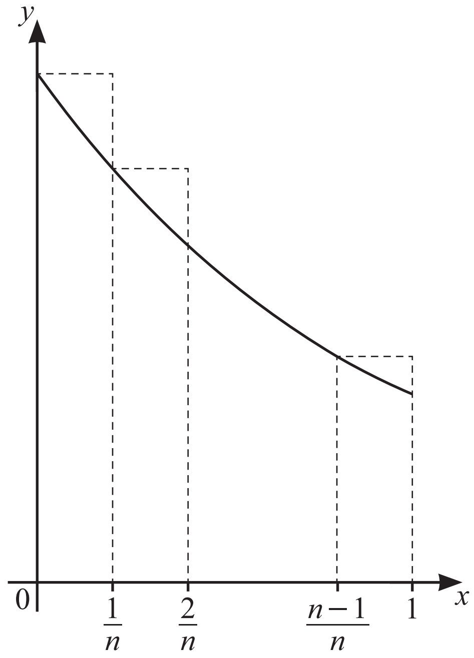

The diagram shows the curve with equation $y = e^{1-x}$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 e^{1-x}\,dx < U_n$, where

$$U_n = \frac{e-1}{n\left(1-e^{-\frac{1}{n}}\right)}.$$

[4]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 e^{1-x}\,dx$. [4]

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(c) Show that $\lim_{n\to\infty}(U_n - L_n) = 0$. [2]

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(d) Use the Maclaurin's series for e^x given in the list of formulae (MF19) to find the first three terms of the series expansion of $z(1-e^{-\frac{1}{z}})$, in ascending powers of $\frac{1}{z}$, and deduce the value of $\lim_{n\to\infty}(U_n)$. [3]

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7 **(a)** Show that $\frac{d}{dx}(\ln(\tanh x)) = 2\operatorname{cosech} 2x$. [3]

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(b) Find the solution of the differential equation

$$\sinh 2x \frac{dy}{dx} + 2y = \sinh 2x$$

for which $y = 5$ when $x = \ln 2$. Give your answer in an exact form. [7]

8 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 7 & 9 \\ 4 & 1 & 7 \end{pmatrix}.$$

(a) Show that the characteristic equation of **A** is $\lambda^3 - 12\lambda^2 + 12\lambda + 80 = 0$ and find the eigenvalues of **A**. [4]

(b) Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{A} + r\mathbf{I},$$

where p, q and r are integers to be determined. [4]

(c) Find a matrix **P** and a diagonal matrix **D** such that $(\mathbf{A}-3\mathbf{I})^4 = \mathbf{PDP}^{-1}$. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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