



Cambridge International AS & A Level

CANDIDATE
NAME



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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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- 1** Find the value of $\int_6^7 \frac{1}{\sqrt{(x-5)^2-1}} dx$, giving your answer in the form $\ln(a + \sqrt{b})$, where a and b are integers to be determined. [4]

[illegible]



(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(2, \frac{1}{2})$.

[4]

[illegible]



3 The curve C has parametric equations

$$x = \frac{1}{2}e^{2t} - \frac{1}{3}t^3 - \frac{1}{2}, \quad y = 2e^t(t-1), \quad \text{for } 0 \leq t \leq 1.$$

Find the exact length of C .

[7]

[illegible]

[illegible]



4 (a) Use de Moivre's theorem to show that

$$\cot 6\theta = \frac{\cot^6 \theta - 15 \cot^4 \theta + 15 \cot^2 \theta - 1}{6 \cot^5 \theta - 20 \cot^3 \theta + 6 \cot \theta}. \quad [6]$$

[illegible]



(b) Hence obtain the roots of the equation

$$x^6 - 6x^5 - 15x^4 + 20x^3 + 15x^2 - 6x - 1 = 0$$

in the form $\cot(q\pi)$, where q is a rational number.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



given that, when $x = 0$, $y = \frac{dy}{dx} = 0$.

[10]

[illegible]



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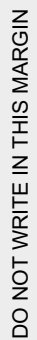
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(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 e^{1-x} dx$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank white space.

(c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

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[3]

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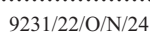
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[3]

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(b) Find the solution of the differential equation

$$\sinh 2x \frac{dy}{dx} + 2y = \sinh 2x$$

for which $y = 5$ when $x = \ln 2$. Give your answer in an exact form.

[7]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal rows, each defined by two parallel dotted lines. The rows are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement. There is no text or other markings on the page.



8 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 7 & 9 \\ 4 & 1 & 7 \end{pmatrix}.$$

- (a) Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 12\lambda^2 + 12\lambda + 80 = 0$ and find the eigenvalues of $\mathbf{A}$. [4]

[illegible]



(b) Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{A} + r\mathbf{I},$$

where p, q and r are integers to be determined.

[4]

[illegible]



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Additional page

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